

Motivation

Trajectory optimization is important in reactor scenario design.

- Trajectory optimization necessitates identification of parameters that influence evolution to unique steady-state solutions.
- In a tokamak, factors like **kinetic profile boundary conditions**, **impurity composition**, and **equilibrium geometry** are likely to affect the evolution.
- Path dependent processes may have a “hidden variable” that influence evolution.

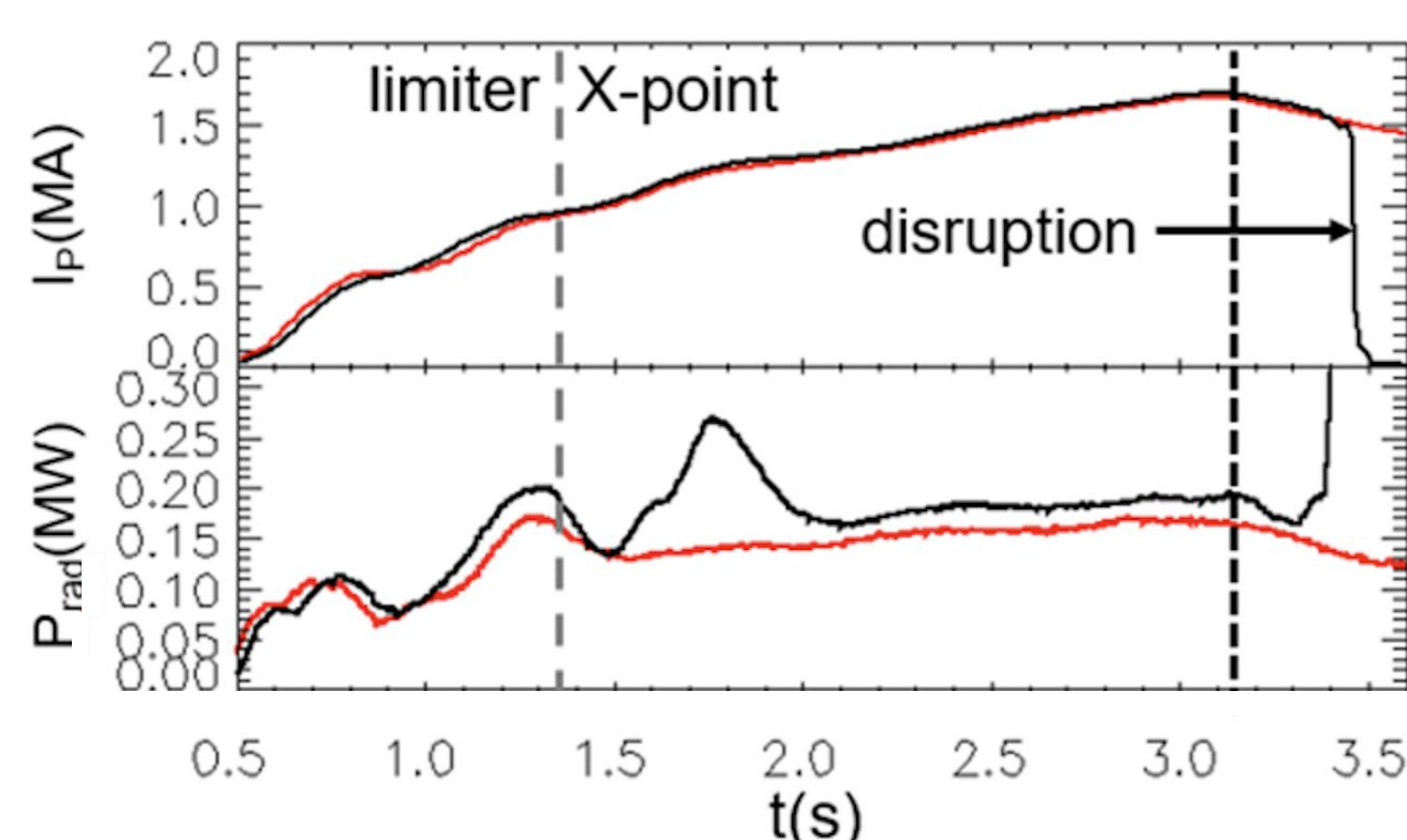


Fig 1 [Left]: Courtesy of JET. The red and black lines are JET shots with identical parameters, but the black shot disrupts, whereas the red shot does not. There is a “hidden variable” at play causing a difference in the two discharges.

Trajectory optimization is useful for **path dependent** parameters.

Methods

TORAX is a differentiable tokamak core transport model [1].

- Torax is a 1D transport solver within a 2D axisymmetric geometry used to evolve equilibria to steady state.

$$(1) \quad \left(\frac{\partial}{\partial t} \right) [n_e V'] = \frac{\partial}{\partial \rho} \left[D_e n_e \frac{g_1}{V'} \frac{\partial n_e}{\partial \rho} - g_0 V_e n_e \right] + V' S_n$$

- QLKNN Turbulent Transport Model
- Sauter Neoclassical Transport Model
- Adaptive Newton-Raphson time stepping

Tokamaker is a time-dependent free-boundary Grad-Shafranov equilibrium solver [2].

- Tokamaker solves the 2D nonlinear Grad-Shafranov equation [eq. 2]

$$(2) \quad \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} = -\mu_0 r^2 \frac{dp}{d\psi} - \frac{1}{2} \frac{dF^2}{d\psi}$$

- Tokamaker was used to create equilibria for q-profile scan with preserved total toroidal current and fixed q values on axis and at edge.

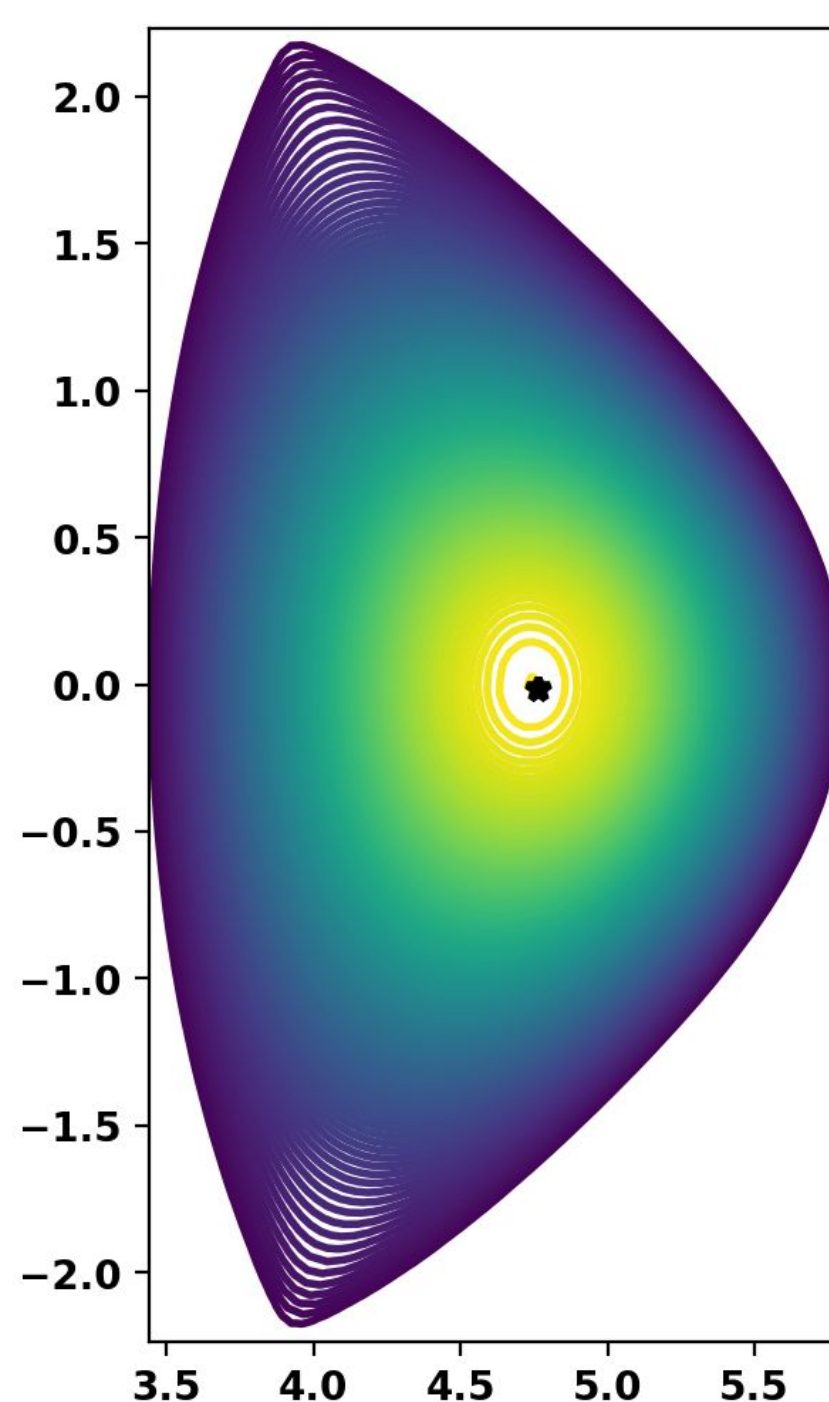


Fig 2: Example contours of approximate solved flux surfaces from Tokamaker G.S. solver.

Which boundary conditions have the largest effect on final core pressure?

ARC and SPARC have different operating core pressures.

The swapping study exchanges estimated **kinetic profile boundary conditions** and **impurity composition** to isolate which parameter accounts for the difference in performance.

Param	1	2	3	4	5
TE_BC	A	S	A	A	A
TI_BC	A	A	S	A	A
NE_BC	A	A	A	S	A
I.C.	A	A	A	A	S

Table 1: Each round, one parameter is changed from the ARC value to the SPARC value. The values are listed in Table 2.

Param	A	S
TE_BC(keV)	0.2	0.34
TI_BC(keV)	0.2	0.69
NE_BC(m ⁻³)	8.4*10 ¹⁹	6.9*10 ¹⁹
I.C.	W, C	Ne, W

Table 2: Key of ARC and SPARC parameter values used in the swapping study.

Isolated Change from ARC to SPARC B.C

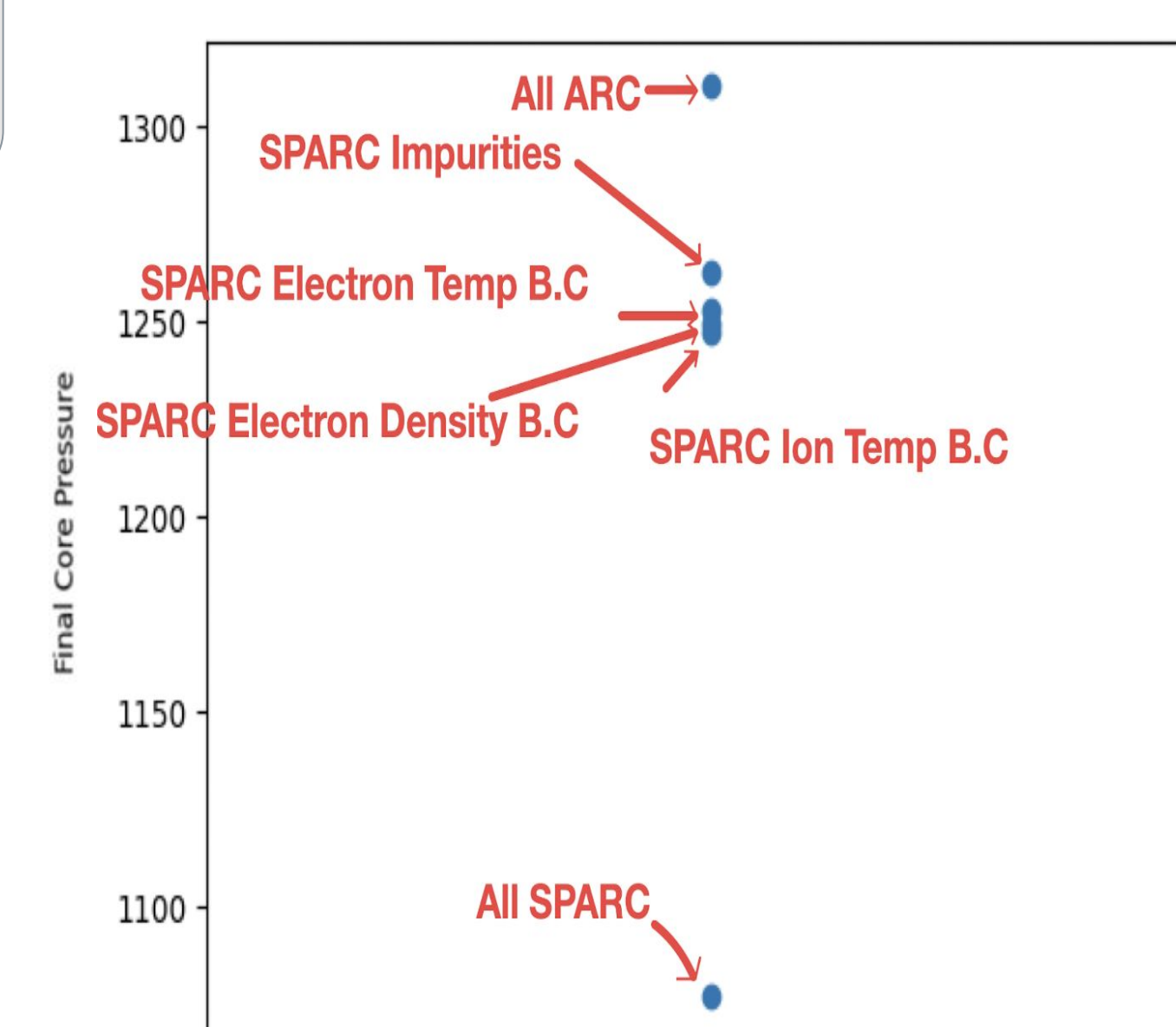


Fig 3: This plot shows the results of systematically replacing one ARC parameter with the SPARC value plotted against the all SPARC and all ARC base case.

Incremental Change from ARC to SPARC B.C

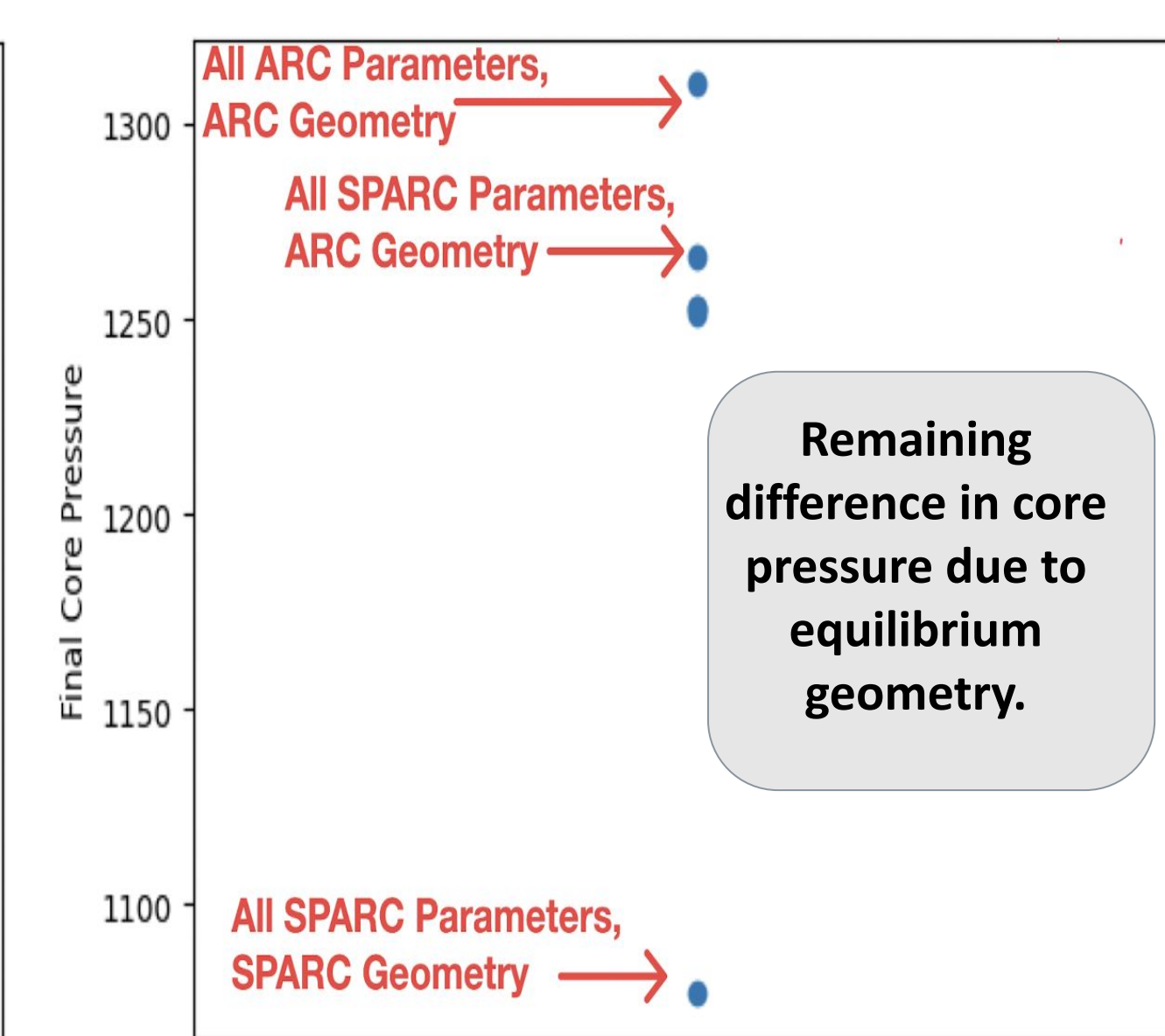


Fig 4: This plot shows an incremental swap from ARC parameters to SPARC parameters against base cases of all ARC and all SPARC equilibria.

- The **equilibrium geometry** is identified to be a large contributor to the difference in ARC and SPARC operating core pressure.

- Check q-profile for **path dependence** by creating a set of equilibria with varying q-profiles and completing a scan in TORAX.

Path Dependence Study of q-Profiles Variations

The FF'(ψ) profile is reshaped by a tunable parameter ε that introduces a flux-localized perturbation, followed by renormalization to preserve the total FF' content.

Modifying FF'(ψ) reshapes the equilibrium magnetic geometry and thus alters q(ψ) through the G.S. solution.

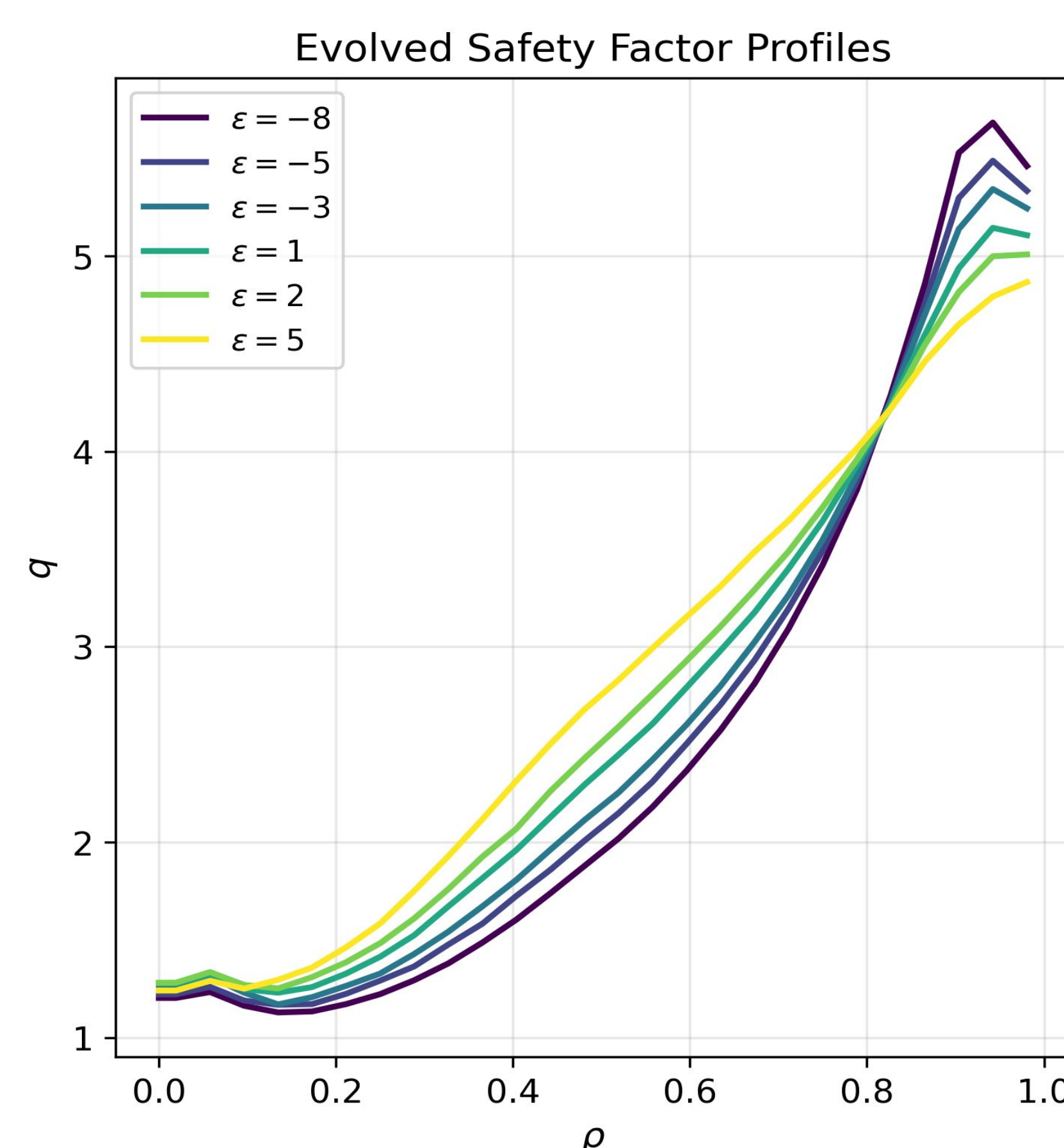


Fig 5: Example of q-profiles used in the path dependence study. Each profile has the same q-profile value on axis and at the edge and the same total current. The profiles are created by a cubic rescaling of the original profile about ρ=0.5. ε=1 refers to the unscaled case.

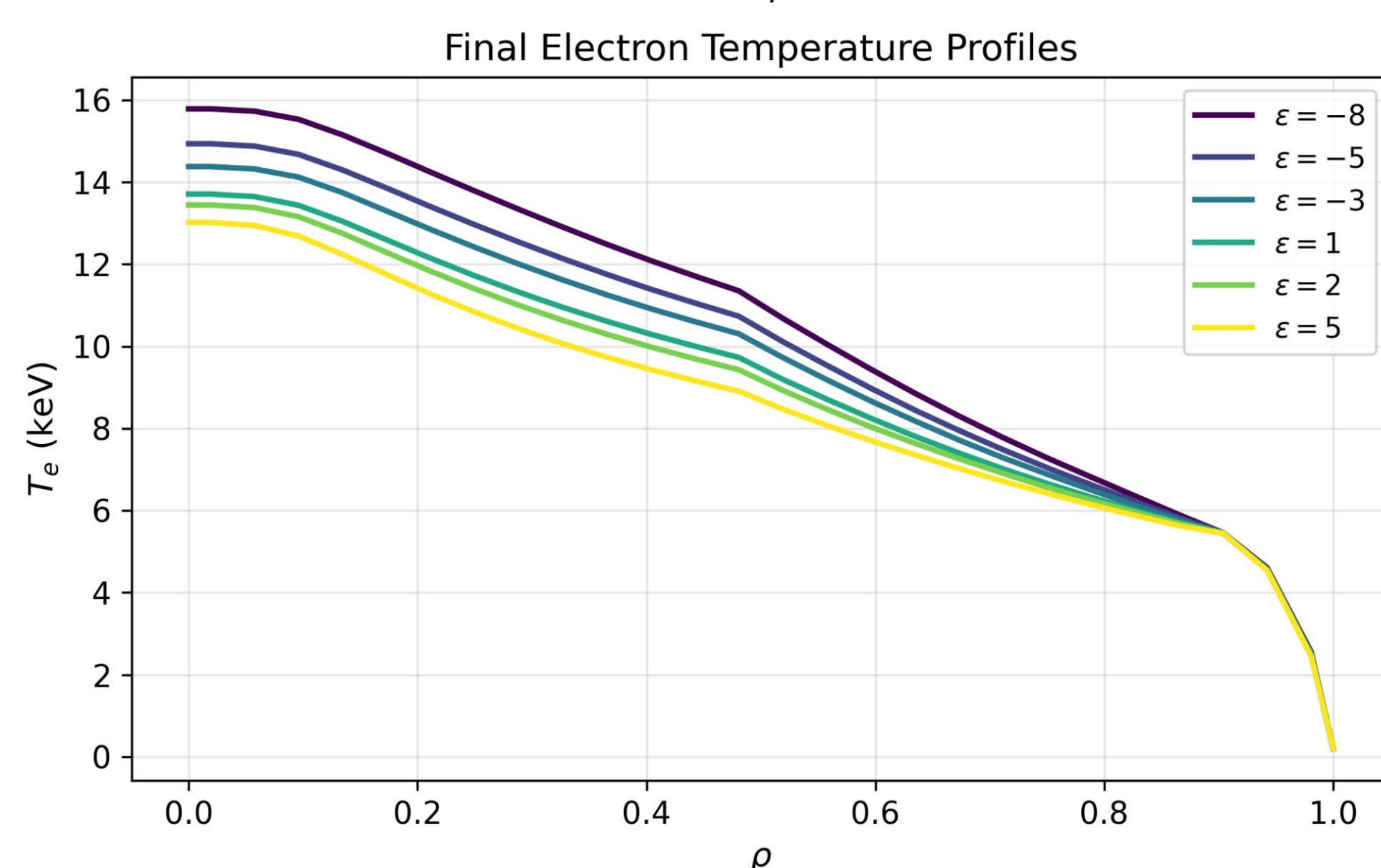
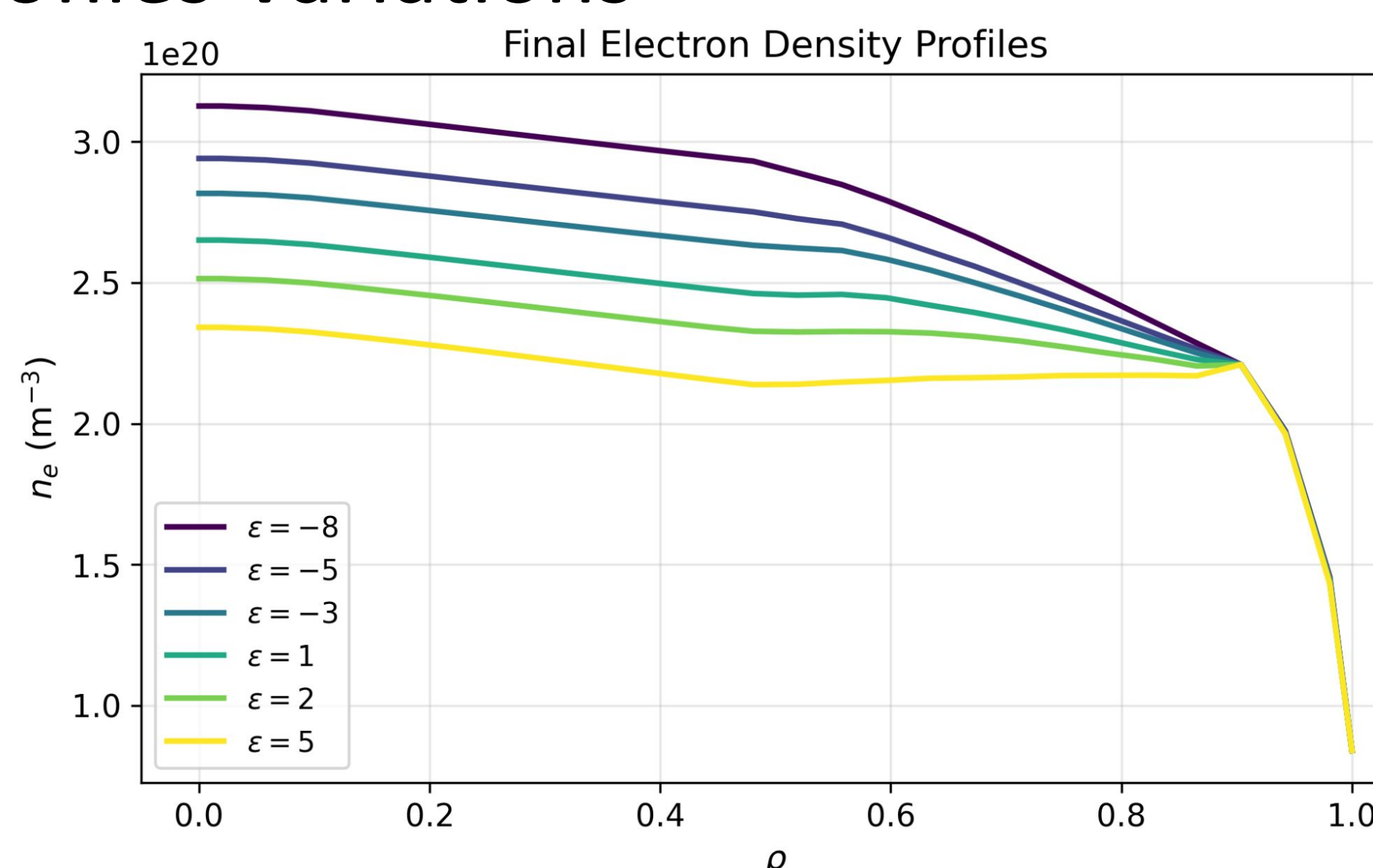


Fig 6, 7: Kinetic profiles resulting from q-profile scan completed in TORAX. The equilibria were evolved to steady state (10 seconds). The base-case is the ε=1 unscaled case.

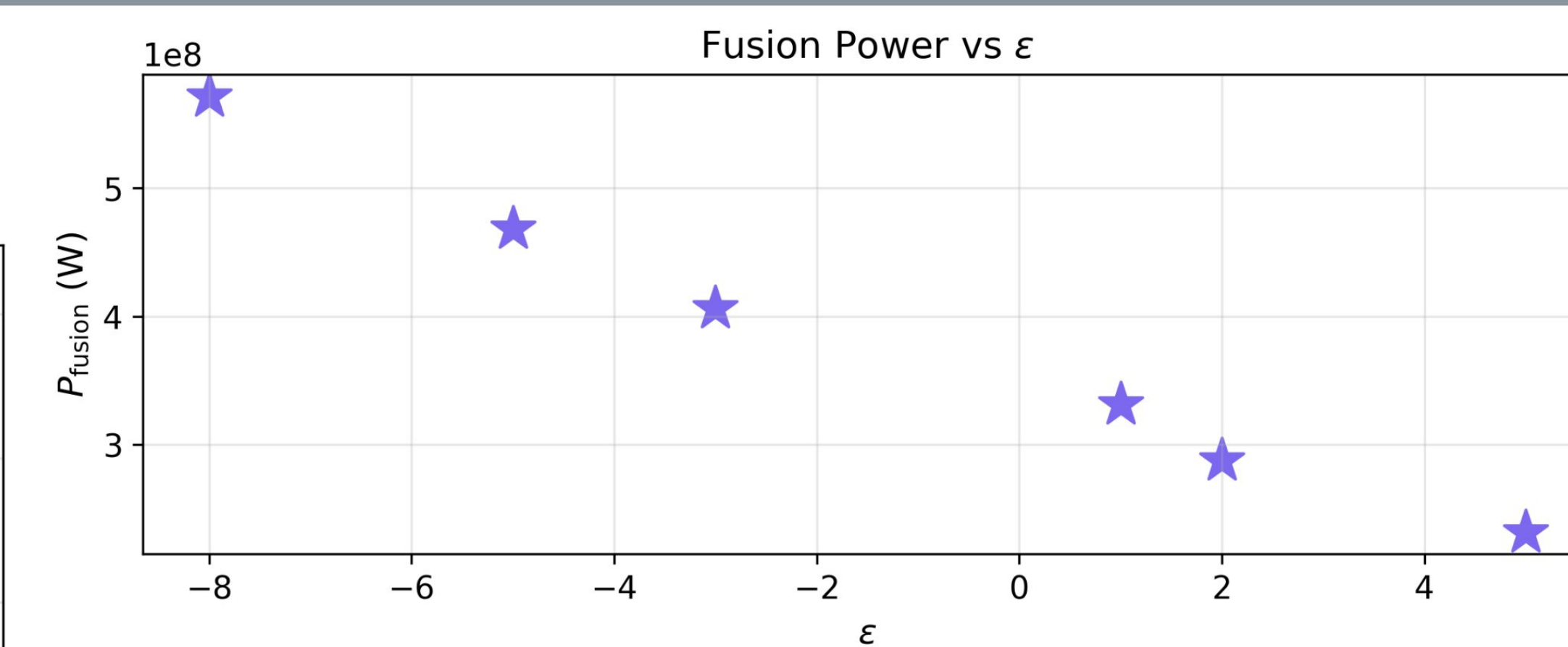


Fig 8: Calculated fusion power for each of the rescaled cases.

TORAX Runs of ε=1 vs ε=-8 cases

Torax simulations of the equilibrium with the steepest, most negative q-profile.

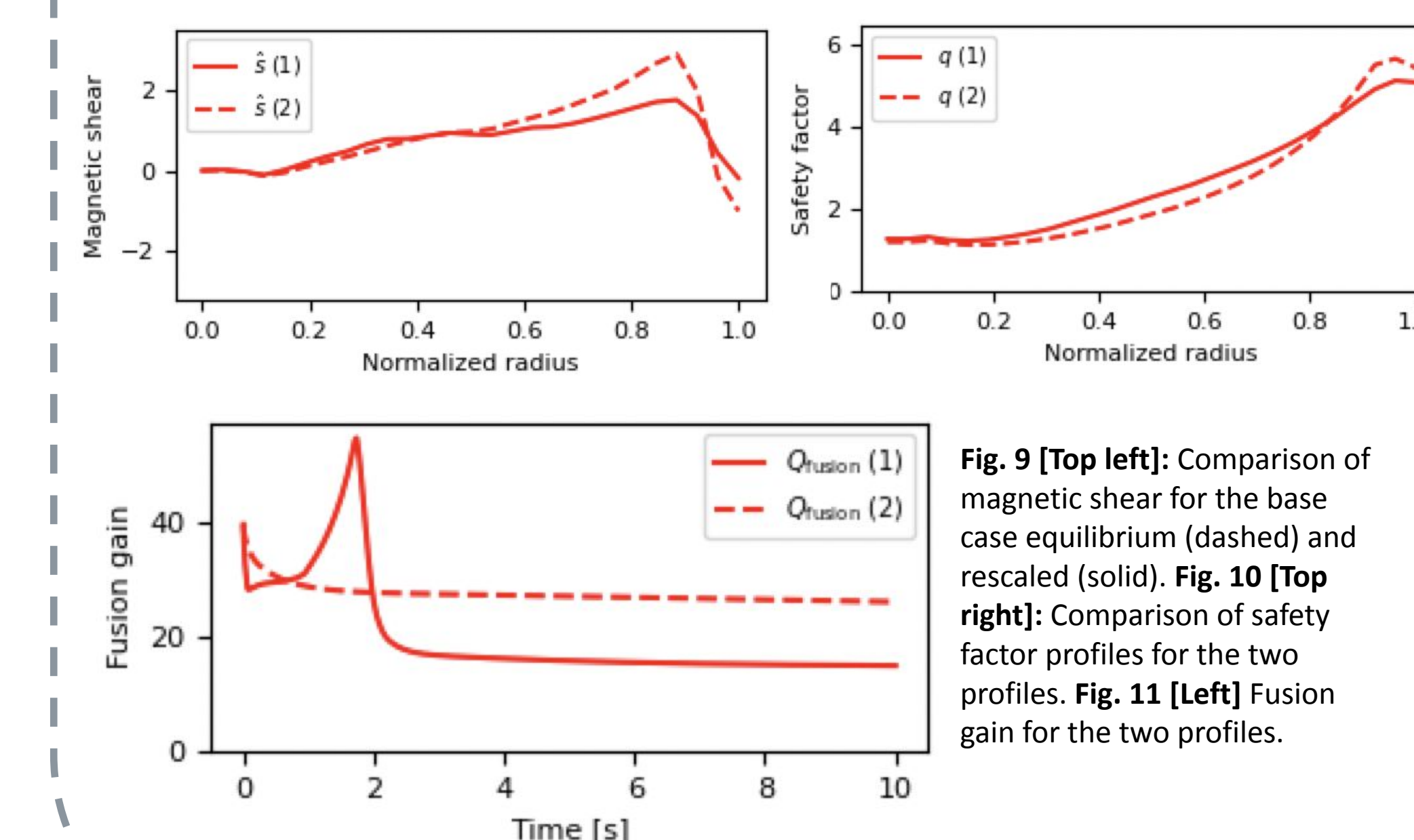


Fig. 9 [Top left]: Comparison of magnetic shear for the base case equilibrium (dashed) and rescaled (solid). **Fig. 10 [Top right]:** Comparison of safety factor profiles for the two profiles. **Fig. 11 [Left]** Fusion gain for the two profiles.

Conclusions and Future Directions

Variations in in safety factor profile result in path-dependent behavior. **The safety factor profile is likely a strong candidate for future tokamak trajectory optimization studies.**

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